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Pathology of massive vector field theory

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Abstract. It is shown that one cannot derive covariant equations for many-point functions of massive Yang–Mills fields by Schwinger's method of functional derivatives or its modifications. The noncovariant terms cannot be removed by modification of the definition of many-point functions.

1. Introduction

After the phenomenological successes of the vector dominance models and the current–field identities (Lee 1968), the problem of the quantization of interacting massive vector (and axial vector) fields has come to receive renewed attention. If one adopts the perturbation theoretical treatment in the interaction representation, one can derive the Feynman rules for a theory involving massive vector (or axial vector) fields with the so-called covariant propagators (Finkelstein *et al* 1971, 't Hooft 1972, Veltman 1970). But if one tries to go beyond the tree approximation in perturbation theory, one encounters unrenormalizable divergences. Finkelstein *et al* (1971) and Nishijima and Watanabe (1972) did not consider the interaction with fermion fields. Nor did they derive equations for many-point functions.

On the other hand, if one wants to explore the possibility of non-perturbative solutions, for example, if one wants to indulge in power counting arguments, one has to derive a chain of equations for many-point functions, without using the free propagators. (Boulware 1970 claimed the impossibility of power counting for massless YM theory in a perturbative approach.)

As an example, let us consider a triplet of massive Yang–Mills fields, W_μ^α, interacting with a doublet of spinor fields, ψ. The invariance group of this model is supposed to be $SU_2 \times SU_2 \times P$, not the Yang–Mills group because of massiveness. In the present note we show that the canonical quantization gives non-covariant equations for many-point functions. (In the present article, the electromagnetic field is not taken into account.)

2. Derivation of equations

The lagrangian (density) for the model under consideration is

$$\mathscr{L}(x) = -\tfrac{1}{4}\sum_{\mu,\nu,\alpha}\left(\frac{\partial W_\mu^\alpha}{\partial x_\nu}-\frac{\partial W_\nu^\alpha}{\partial x_\mu}-g\sum_{\beta,\delta}\epsilon^{\alpha\beta\delta}W_\mu^\beta W_\nu^\delta\right)^2(x)-\tfrac{1}{2}M^2\sum_{\mu,\alpha}W_\mu^\alpha W_\mu^\alpha(x)$$

$$-\sum_{\mu,\kappa}\bar{\psi}^\kappa\left(\gamma_\mu\frac{\partial}{\partial x_\mu}-m\right)\psi^\kappa(x)-\mathrm{i}f\sum_{\kappa,\lambda,\alpha,\mu}\bar{\psi}^\kappa\gamma_\mu\tau^\alpha_{\kappa\lambda}W_\mu^\alpha\psi^\lambda(x)+\mathscr{L}^{(\mathrm{s})}(x) \tag{1}$$

$$\mathscr{L}^{(\mathrm{s})}(x) = -\sum_{\alpha,\mu}W_\mu^\alpha\xi_\mu^\alpha-\sum_\kappa(\bar{\psi}^\kappa\eta^\kappa+\bar{\eta}^\kappa\psi^\kappa).$$

Here ξ, η and $\bar{\eta}$ are the fictitious external sources introduced to derive the equations for the many-point functions by Schwinger's (1951) method (see also Symanzik 1954, Umezawa 1956, Rzewuski 1969).

The many-point functions are defined as follows:

$$G^{\alpha_1\alpha_2\ldots\alpha_l}_{\mu_1\mu_2\ldots\mu_l}(x_1,x_2,\ldots,x_l;y_1,y_2,\ldots,y_l;z_1,z_2,\ldots,z_l:\xi,\eta,\bar{\eta})$$

$$\equiv\frac{\delta^l}{\delta\xi^{\alpha_1}_{\mu_1}(x_1)\ldots\delta\xi^{\alpha_l}_{\mu_l}(x_l)}\frac{\delta^l}{\delta\bar{\eta}(y_1)\ldots\delta\eta(y_l)}\frac{\delta^l}{\delta\eta(z_1)\ldots\delta\eta(z_l)}$$

$$\times\langle 0|U[\sigma,\sigma':\xi,\eta,\bar{\eta}]|0\rangle \tag{2}$$

where

$$\mathrm{i}\frac{\delta\langle 0|U[\sigma,\sigma':\xi,\eta,\bar{\eta}]|0\rangle}{\delta\sigma(x)} = \mathscr{L}^{(\mathrm{s})}(x)\langle 0|U[\sigma,\sigma':\xi,\eta,\bar{\eta}]|0\rangle. \tag{3}$$

The relevant equal-time commutation relations that are derived by the canonical quantization of the lagrangian (1) are

$$\left[W_k^\alpha(\boldsymbol{x},x_0),\left(\frac{\partial W_l^\beta}{\partial x_4}-\frac{\partial W_4^\beta}{\partial x_l}-\mathrm{i}g\epsilon^{\beta\gamma\delta}W_4^\gamma W_l^\delta\right)(\boldsymbol{x}',x_0)\right] = \mathrm{i}\delta^{\alpha\beta}\delta^3(\boldsymbol{x}-\boldsymbol{x}')$$

$$[W_4^\alpha(\boldsymbol{x},x_0),W_k^\beta(\boldsymbol{x}',x_0)] = \mathrm{i}M^{-1}\left(\delta^{\alpha\beta}\frac{\partial}{\partial x_k}-\mathrm{i}g\epsilon^{\alpha\beta\gamma}W_k^\gamma(x)\right)\delta^3(\boldsymbol{x}-\boldsymbol{x}')$$

$$[W_4^\alpha(\boldsymbol{x},x_0),\psi(\boldsymbol{x}',x_0)] = \mathrm{i}fM^{-2}\tau^\alpha\psi(\boldsymbol{x},x_0)\delta^3(\boldsymbol{x}-\boldsymbol{x}') \tag{4}$$

$$[W_4^\alpha(\boldsymbol{x},x_0),\bar{\psi}(\boldsymbol{x}',x_0)] = \mathrm{i}fM^{-2}\bar{\psi}(\boldsymbol{x},x_0)\tau^\alpha\delta^3(\boldsymbol{x}-\boldsymbol{x}')$$

$$\{\psi(\boldsymbol{x},x_0),\bar{\psi}(\boldsymbol{x}',x_0)\} = \mathrm{i}\gamma_4\delta^3(\boldsymbol{x}-\boldsymbol{x}').$$

In order to derive an 'equation' for the $W\psi\bar{\psi}$ vertex part, let us operate

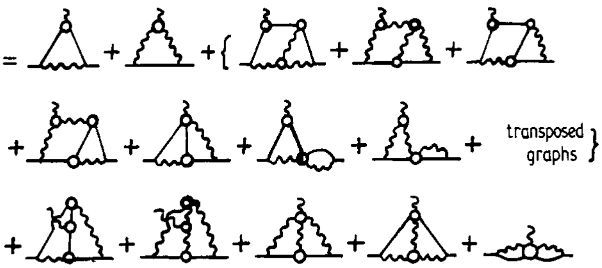
$$\gamma_\mu\frac{\partial}{\partial y_\mu}-m-\mathrm{i}f\gamma_\mu\tau^\alpha\frac{\delta}{\delta\xi^\alpha_\mu(y)}$$

and

$$-\gamma_\nu\frac{\overleftarrow{\partial}}{\partial z_\nu}-m-\mathrm{i}f\gamma_\nu\tau^\beta\frac{\overleftarrow{\delta}}{\delta\xi^\beta_\nu(z)}$$

upon $G^{\rho}_{\lambda}(x;y;z:\xi,\eta,\bar{\eta})$ and put $\xi\equiv 0,\eta\equiv 0,\bar{\eta}\equiv 0$. Then we get

$$\int \mathrm{d}x' G^{\rho\sigma}_{\lambda\nu}(x-x')\Gamma^{\sigma}_{\nu}(x',y,z)$$
$$= f G^{\rho\sigma}_{\lambda\nu}(x-y)\gamma_{\nu}\tau^{\sigma}\delta^{4}(y-z)+\int \mathrm{d}x' G^{\rho\sigma}_{\lambda\nu}(x-x')\Lambda^{\sigma}_{\nu}(x',y,z)$$
$$+fM^{-2}\delta_{\lambda 4}\gamma_{4}\tau^{\rho}\delta^{4}(x-y)\delta^{4}(x-z)+fM^{-2}\gamma_{4}\tau^{\sigma}G^{\rho}_{\lambda}(x;y;y)\gamma_{4}\tau^{\sigma}\delta^{4}(y-z). \quad (5)$$

$\Lambda^{\sigma}_{\nu}=$ 

(6)

The last two terms on the right-hand side of equation (5), due to the noncommutativity of W_4 and ψ (and $\bar{\psi}$), are not covariant. Substituting the right-hand side of (5) for the $W\psi\bar{\psi}$ vertex parts in equation (6), one finds, without the Landau approximation (Landau *et al* 1954) and like for the vertex parts, for example,


(7)

where the 'equations' for

$$G^{\alpha_1\alpha_2\alpha_3}_{\mu_1\mu_2\mu_3}(x_1,x_2,x_3;\phi;\phi:\xi\eta\bar{\eta})$$

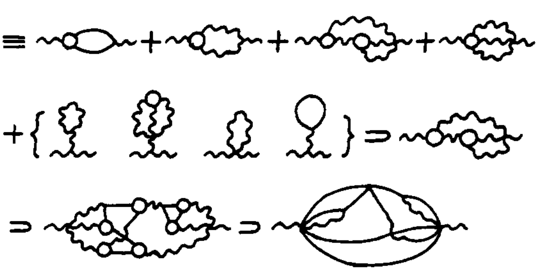
and

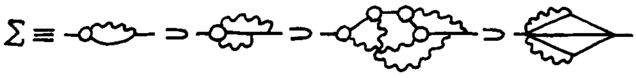
$$G^{\alpha_1\alpha_2\alpha_3\alpha_4}_{\mu_1\mu_2\mu_3\mu_4}(x_1,x_2,x_3,x_4;\phi;\phi:\xi\eta\bar{\eta})$$

which are also non-covariant are used. Here, ϕ stands for absence of variables of the corresponding class. The last diagram contains six-way junctions

,

though the lagrangian (1) does not contain an interaction term $g^2(\epsilon^{\alpha\beta\gamma}\bar{\psi}\gamma_4\tau^{\beta}\psi W^{\gamma}_{k})^2$. Few other examples of inclusions are

$\Pi\equiv$ 
(8)



(9)

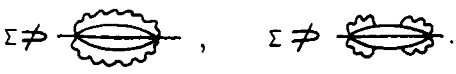
where Π and Σ are the self-energy parts of the Yang–Mills fields and of the spinor fields, respectively. The contribution of the graphs in { } of (8) is constant and must be removed by mass renormalization.

On the other hand, one finds, for example

(10)

but

(11)



(12)

It should be noted that Σ does not include any diagram with six-way junctions at both ends. From the above examples, it is obvious that the diagrams containing six-way junctions cannot be removed by introduction of non-covariant counter terms into the lagrangian (or hamiltonian).

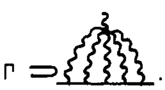
If one tries to redefine the $W\psi\bar{\psi}$ vertex part by the relation

$$\left.\frac{\delta^3\langle 0|U[\sigma,\sigma':\xi,\eta,\bar{\eta}]|0\rangle}{\delta\xi^{\alpha}_{\mu}(x)\delta\bar{\eta}(y)\delta\eta(z)}\right|_{\xi\equiv\eta\equiv\bar{\eta}\equiv 0}$$

$$= \int dx'\,dy'\,dz'G^{\alpha\beta}_{\mu\nu}(x,x';\phi;\phi:000)G(\phi;y;y':000)\Gamma^{\beta}_{\nu}(x',y',z')G(\phi;z';z:000)$$

$$-M^{-2}\delta_{\mu 4}\int dy'\,dz'G(\phi;y;y':000)\Gamma^{\alpha}_{4}(x,y',z')G(\phi;z';z:000) \qquad (13)$$

in order to incorporate the idea of the so-called covariant propagator, one finds new non-covariant terms, for example,



(14)

In other words, *ad hoc* surgery upon many-point functions does not cure the disease but gives rise to metastasis.

3. Concluding remarks

As has been seen above, Schwinger's (1951) method gives rise to non-covariant terms in the 'equations' for the many-point functions, if applied to massive Yang–Mills fields interacting with spinor fields. Modifications of the definition of the generating functional as have been so far made in other contexts are either for deriving perturbative Feynman rules or do not apply to the massive Yang–Mills fields at all.

Ad hoc redefinition of propagators etc is also useless, because such a recipe introduces new non-covariant terms. Attempts to remove the new non-covariant terms generate further non-covariant terms. Such procedures go *ad infinitum* without curing the disease.

The following questions have not been answered: (i) whether covariant on-shell S matrix elements can be derived from non-covariant 'equations' for (off-shell) many-point functions; (ii) whether the above trouble is a manifestation of Haag's theorem (Haag 1955) that is, of the non-equivalence of the interaction representation and the Heisenberg representation.

The purpose of this article has been to show that neither straightforward application nor any conceivable modification thereof yields a system of formally covariant equations for many-point functions of massive Yang–Mills fields interacting with fermion fields. As we did not resort to Källén–Lehmann representation of two-point functions, formally covariant contributions of intermediate states with negative norms are not excluded. The discrepancy of 't Hooft's (1972) conclusion and ours seems to be due to the fact that he uses Feynman rules in terms of free propagators without referring to the notion of the physical (Heisenberg) vacuum, while we avoid interaction representation of any sort. If one wants to achieve a meaningful theory of interacting fields without ignoring Haag's theorem, one ought to decline to resort to the interaction representation.

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